| Titolo del modulo | **FUNCTIONS** |
|---|---|
| **classe** | Seconda (fino a pag. 8)<br>Terza ( tutto) |
| **scuola** | I.T.I. A. Malignani – Udine |
| **livello linguistico** | A2 - Pre-intermediate |
| **punto del programma** (eventuali prerequisiti) | Operazioni con gli insiemi - prodotto cartesiano- Insiemi numerici |
| **contenuti disciplinari** | Relazioni e funzioni |
| **numero di ore** | 6 ore in seconda<br>10 ore in terza |
| **periodo** | novembre – dicembre |
| **materiale** (libri, software, DVD, videocassette, fotocopie…) | Fotocopie per la spiegazione<br>Libro di testo per rinforzare le conoscenze e arricchire le competenze<br>Software didattico Epsilon dal quale sono state estratte alcune parti |
| **supporti** (laboratorio, lavagna luminosa, video….) | Lavagna<br>Laboratorio per visualizzare grafici |
| **compresenza con l'insegnante d'inglese(?)** | Impossibile a causa dell'orario scolastico |

# SCHEMA MODULO

I) <u>ATTIVITA' DI BRAINSTORMING</u> (tempo 5-10 minuti). Si parte con:

a. *warm-up questions*
b. parole chiave (scritte alla lavagna)
c. immagine
d. riassunto della/e lezione/i precedente/i fatta dagli studenti
e. Ripasso prodotto cartesiano e coppie ordinate

II) <u>PIANO DELLE LEZIONI</u>.

Le lezioni si svolgeranno seguendo gli appunti che saranno forniti agli studenti all'inizio delle varie unità orario. Gli studenti seguiranno le istruzioni e risponderanno alle domande che via via troveranno o che l'insegnante farà loro.

## FUNCTIONS

**Exercise 1**

Why fly to Milan in January?

Several people arriving at Milan airport from London were asked the purpose of their visit.

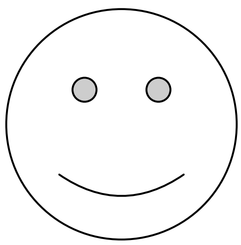
| **A / People** | **B / Reasons** |
|---|---|
| Joanne | Skiing |
| David | Returning home |
| Jonathan | To study abroad |
| Louise | Business |
| Paul | Visiting friends |
| Shamaila | |
| Karen | |

Match each person belonging to the set of the people A with a purpose belonging to B.
Compare with your neighbour.

**Note :** Is it possible that some reasons couldn't be matched with people?
How many pairs has the cartesian product AxB ? How many pairs have you found ?

✧ A rule which associates two( not empty) sets A, B of items is a **MAPPING** or **MAP** and it is a subset of AxB.

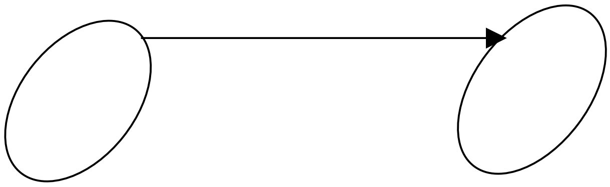
**Exercise 2**
The sets A and B are the same and their elements are the numbers on a clock: A=B={1,2,3,4,5,6,7,8,9,10,11,12}



Map around the **face of the clock** with the relation $n \to n+5$ until you reach again 1;(the arrow indicates that a number n is matched with n+5, e.g. 1 is matched with 6).

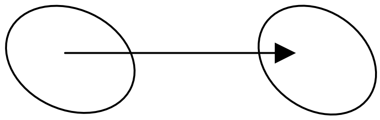
Complete the table and the Venn diagrams below and compare your notes with a partner .

| n | 1 | 2 | 3 | 4 | | | | | | | | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| n+5 | | 7 | | | | | | | | | | |



**Exercise 3**

Let A = {0,1,-1,2,3,-3} and B = {0,1,2,3,4,5}
and n → $n^2$ the relation.
Using Eulero-Venn diagrams, match the corresponding numbers.



✧ The set of all possible elements ( in the left set A) or inputs, that have an image into B , is called **domain** of the map and we indicate it as **D**.
In this example **D** = { 0, . , . , . , . , . , . }

✧ The set of all **images** or outputs ( in the set on the right-hand ) is called **range** and we indicate it as **R**.
In the example above it is R = { ., 1, .}

Now look back at all the previous examples and find out the domain and the range of each map.

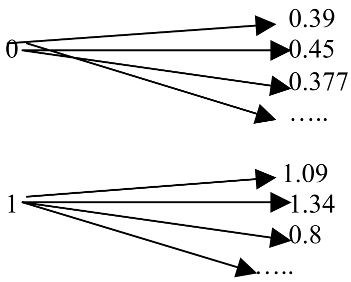
Ex 1. **D** = {…. … } and **R** = { …… … …. }

Ex 2. **D** = { } and **R** = { }

OTHER EXAMPLES

**Exercise 4**
**A = Z** ( set of natural numbers)
**B = R** ( set of real numbers )

**INPUTS** **IMAGES / OUTPUTS**




2
…. ……….

General rule :
Rounded whole number → Unrounded real numbers
Domain = **D** = …
Range = **R** = ..

**Exercise 5**
**N** ------------→ **N**

| | |
|---|---|
| 0 | 0 |
| 1 | |
| 2 | 1 |
| 3 | |
| 4 | 2 |
| 5 | |
| 6 | 3 |

Match each possible element in the first column with the corresponding element in the second column following the general rule : n → n/2
Domain = **D** = …
Range = **R** = ..

Mappings can be:

- one-to-one if each one value of x maps onto exactly one value of y;
- many-to-one if two, or more, values of x map onto exactly one value of y;
- one-to-many if each one value of x maps onto two, or more, values of y;

**!!!!** Mappings which are one-to-one or many-to-one are of particular importance, since in this case there is just one possible image for each input. Mappings of these types are called **Functions**.

✦ A **function** is a rule ( or re that maps each element in a given not empty set A, onto just one element ( or image) in an other set B, not empty too.

- To concrete the meaning of the definition of function we can use such an analogy: Think of the first set A as the set of children and the second set B as the set of mothers. So:

1. you may not see any child without mother 2. But a mother may have no children
3. A child may have only one mother 4. But a mother may have lots of children.

Exercise 6

Decide if the maps given in the previous examples are functions or not. Discuss it with your desk mate.

There are several different but equivalent ways of writing down a function.
For example, the function that maps x onto $x^2$ can be written in any of the following ways :

$y = x^2$ $f(x) = x^2$ $f : x \rightarrow x^2$ $f : A \rightarrow B$, $x \rightarrow x^2$

Read ' f of x equal to x² '

Read this as ' f maps x onto x² '

If $f(x)= - x^2+2x$ calculate: $f(-1)=$ , $f( x+1)=$ , $f(-x)=$

✲ A **function** is **injective** ( or **one to one** in same cases ) if different elements of the domain are paired with different elements of the range.
For example the mapping associating oldest sons to fathers is injective when A and B are the set of all men;

$f(x) = x^2$ is not injective on the reals since $f(x)=f(-x)$, but it is injective on the positive real numbers.

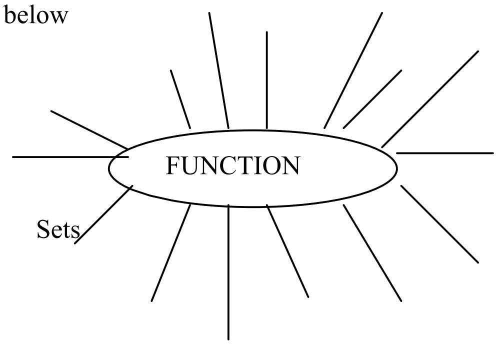
✲ A **function** that maps A onto B is **surjective** **(onto)** if the range (or set of all images) is equal to B; it means that every element of the set B is the image of at least one element of the domain.

✲ A function is **bijective** if it is both injective and surjective.

Examples:
( students are asked to suggest examples of functions one to one, onto or both)

FOR REVIEW and CLASS DISCUSSION

Complete the spidergram below

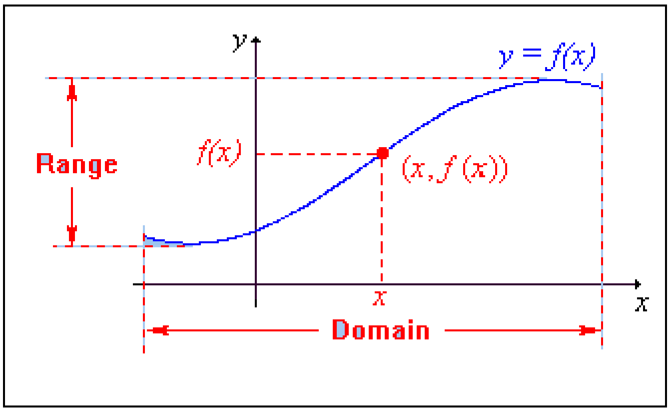


Fill in the gaps:

A …………………… from A to B is any subset of ….. ………… ……… AxB.
In the …………………. pair (a , b ), a is the …….. ……………….. and b is the second………………
A…………………… from A to B is a …….. that maps ……..element in a given set …..onto just ………element, (or …………), in the set ….., provided that the two sets are not………
The domain is the set of …… …………...belonging to the set ….,that have an ……….onto ……
The range is the set of all…………belonging to the set …..
A function is …………….. if it pairs different elements of the ………….. with different elements of the …………
A function that ……….A onto B is……………. **(onto)** if the ………… (or set of all images) is equal to ……

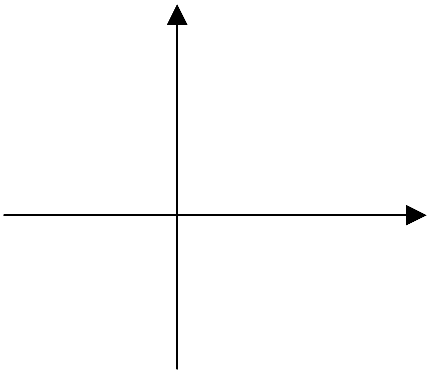
**When a function is given using a mathematical expression, it is useful to have a graphical representation of it as this helps to visualise the behaviour of the function.**

**The graph of a function $f(x)$ is the set of all points of the form $(x, f(x))$ in the coordinate plane, where $x$ is in the domain of $f$.**




When you draw a graph of a mapping or of a function, the x coordinate ( independent variable) of each point is an input value, the y coordinate (dependent variable) is the corresponding output value( or Image).
The table below shows this for the mapping $x \to x^2$ ( R-->R) or $y = x^2$ . Plot the resulting points on the cartesian plane and draw the graph.

| Input (x) | Output (y) | Point plotted |
|---|---|---|
| -2 | 4 | (-2;4) |
| -1 | 1 | (-1;1) |
| 0 | 0 | ( 0;0) |
| 1 | 1 | ( 1;1) |



Example :
A purple-nosed buffalo is walking at 2 m/sec; what distance S, does it cover within T = 1 sec? S=….; T = 2 sec? S=…… ; T = 10 sec? S=……… etc..
The general rule is S = ….*….

You can easily draw the graph of this function using this method :
1. Choose three or more values of T and draw up a wee table

2. Work out the S values
3. Plot the coordinates on the cartesian plane and draw the line ( straight line in this case! )

Exercise 7.
Sketch the graph of y = 3x +2, from D to R, when the domain D is :(a) R (b) Z (c) N and compare with your friend

!! This example illustrates the importance of knowing the domain.

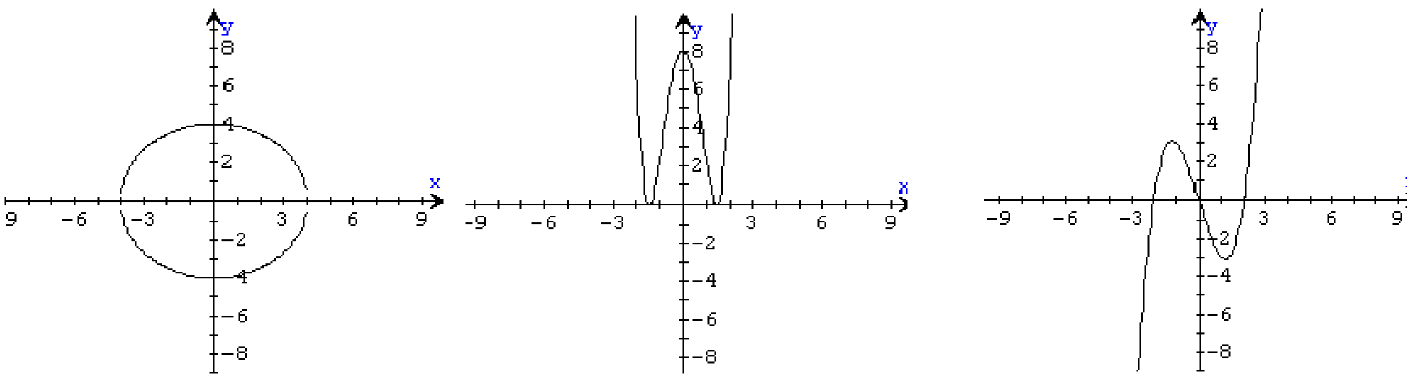
## Keep in Mind:

! The domain of a function is the set of x values that are allowed and the range is the corresponding set of y values.

!! If the map is a function there is one and only one image for every input, or x value, in the domain.

!!! Consequently the graph of a function is a simple curve or line going from left to right with no doubling back and each vertical line crosses the graph only once ( **vertical line test** ).

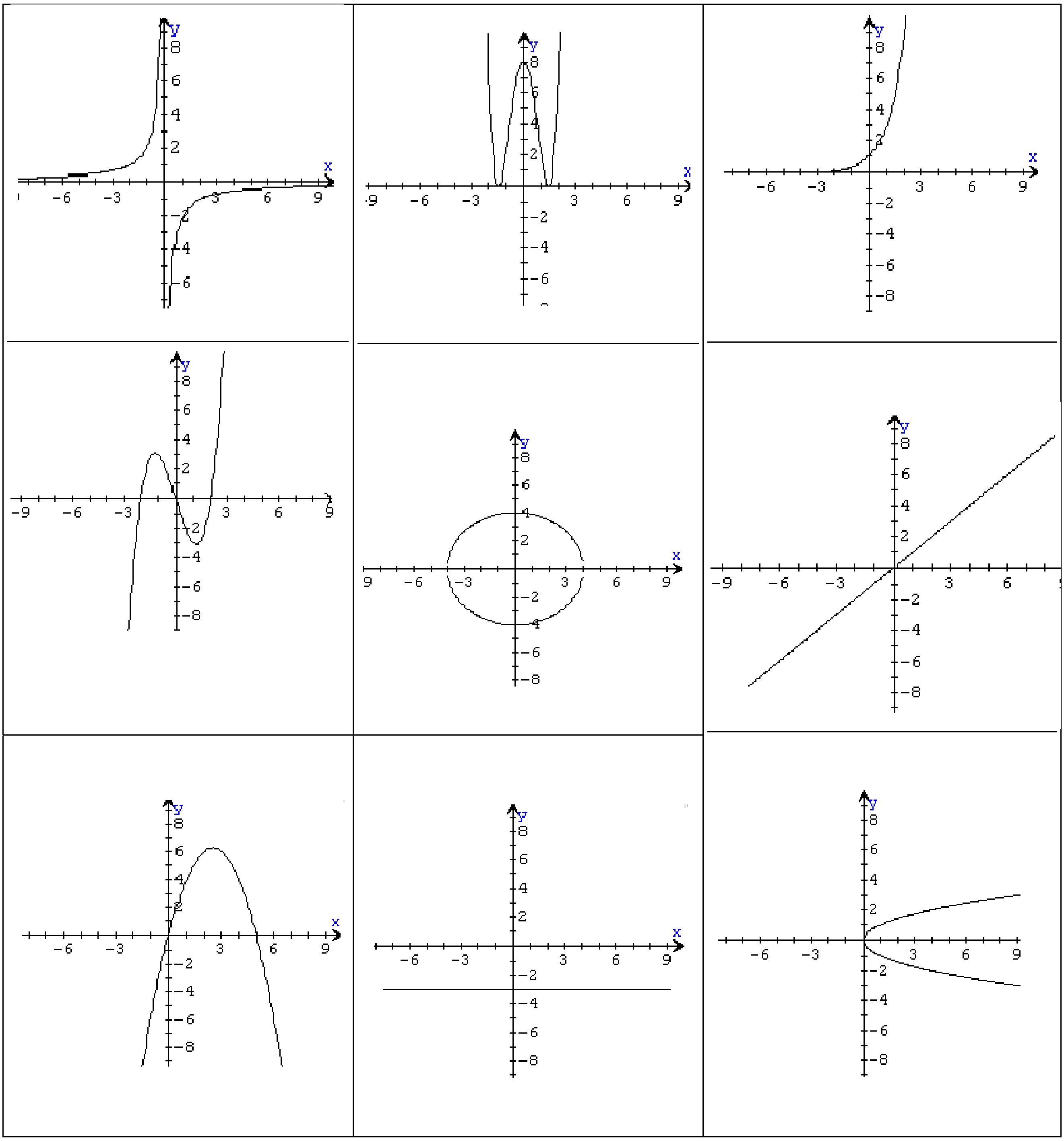
## Examples:




Which of these graphs illustrate functions?
Discuss with your partner and explain why.

**EXSERCISE-REVISION**

Describe each of the following mappings as either one-to-one, many-to-one, one-to-many and say whether it represents a function; if the answer is yes, say if it is injective, surjective or bijective.



Test for students attending a second class….

Functions can be combined to form new functions. We can add, subtract, multiply and divide functions in much the same way as we add, subtract, multiply and divide real numbers. Given two functions $f$ and $g$ with domains $A$ and $B$, we define $f+g$ so that the image for each value of $x$ is the sum of the corresponding images, that is

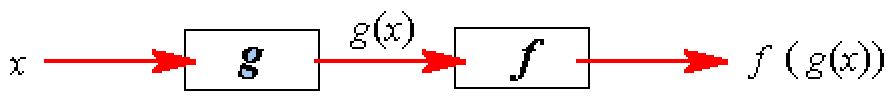
$$(f+g)(x) = f(x) + g(x)$$

This obviously makes sense only if $x$ belongs to the domain of $f$, $A$, and also to the domain of $g$, $B$.
Similarly, we have

$$(f-g)(x) = f(x)-g(x) \qquad x \in A \cap B$$
$$(f\,g)(x) = f(x)\,g(x) \qquad x \in A \cap B$$
$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \qquad x \in A \cap B \text{ and } g(x) \neq 0$$

## Composition of functions

Composition is another way of combining two functions $f$ and $g$. The representation of a function as a machine helps us to understand this concept. The output of the function $g$ is the input for the function $f$ :




For each value $x$ we have a final output $f\,(g\,(x))$.

Given two functions $f$ and $g$, the composite function $f \circ g$, is defined as

$$f \circ g\,(x) = f(\,g(x))$$

Composition makes sense for any value $x$ in the domain of $g$ such that the value $g(x)$ is in the domain of $f$.

**Example:**

Let $f(x) = x^2$ and $g(x) = \sqrt{1-x}$.

Find $f \circ g$ and $g \circ f$ and their domains.

$$(f \circ g)(x) = f(\sqrt{1-x}) = \left(\sqrt{1-x}\right)^2 = 1-x$$

$g$ is defined for $x \leq 1$, while $f$ is defined for any value.
Therefore $f \circ g$ is defined for $x \leq 1$.

$$(g \circ f)(x) = g(x^2) = \sqrt{1-x^2}$$

$f$ is defined for any value, but $g$ is defined only for $x \leq 1$.
So we need $x^2 \leq 1$, which gives $-1 \leq x \leq 1$. Therefore the domain of $g \circ f$ is $-1 \leq x \leq 1$.

This example shows that in general $f \circ g \neq g \circ f$.

## Inverse functions

Two functions are inverses of each other if each of them undoes what the other one does. For example, look at the function

$$f(x) = 2x + 3$$

What does this function do to a value of $x$? First, it multiplies this number by 2 and then it adds 3 to the result. If we want to undo the action of $f$ we need first to subtract 3 and then divide by 2. So the inverse function of $f$, which we denote by $f^{-1}$, is

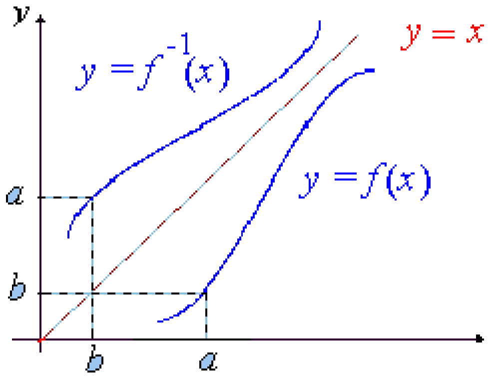
$$f^{-1}(x) = \frac{x-3}{2}$$

Given a function $f$ with domain $A$ and range $B$, $f^{-1}$ is the inverse function of $f$ if $f^{-1}(f(x)) = x$ for every $x$ in $A$ and $f(f^{-1}(x)) = x$ for every $x$ in $B$.
If a function $f$ has an inverse, we say that $f$ is invertible.

## Graph of inverse functions

It is easy to obtain the graph of $f^{-1}$ if the graph of $f$ is known. This is because if a point $(a,b)$ belongs to the graph of $f$, then the point $(b, a)$ belongs to the graph of $f^{-1}$. The point $(b, a)$ is the reflection of the point $(a,b)$ in the line $y = x$, so

> the graph of $f^{-1}$ is the reflection of the graph of $f$ in the line $y = x$.




## Not every function has an inverse !

Take for example the function $f(x) = x^2$; 4 belongs to the range of $f$, but how do we define $f^{-1}(4)$? Should it be 2 or −2 ? We find ourselves in this quandary because the squaring function is not one to one: there are pairs of values with identical images.

So if we want to undo what a function does, each value in the range has to be the image of only one value in the domain. We state this as follows:

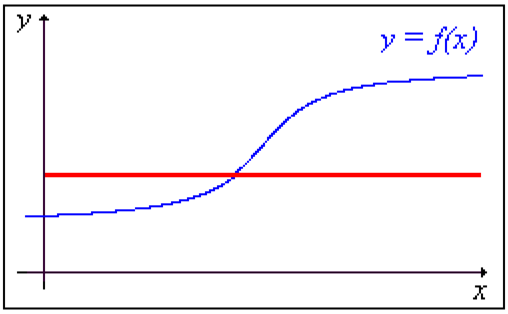
> A function is invertible if and only if it is one to one.

Note that if we restrict the domain of the squaring function to $x \geq 0$, then the function is one to one and the square root reverses its action.

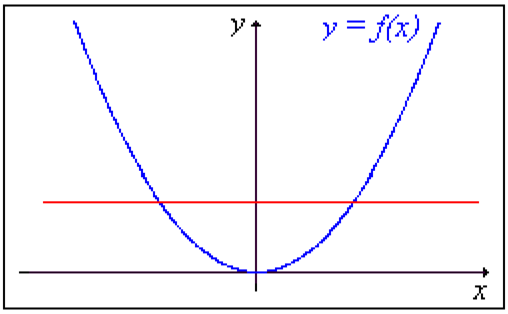
For $f(x) = x^2$, $x \geq 0$ we have $f^{-1}(x) = \sqrt{x}$

## Graphical test for a one-to-one function:

Looking at the graph of a function, this will have an inverse only if for each value $y$ in the range of $f$ there is a unique $x$. That is, only if each horizontal line intersects the graph at most once.




Each horizontal line intersects the graph only once, therefore the function has an inverse in all its domain.




Horizontal lines intersect the graph twice, so this function doesn't have an inverse in all its domain. But an inverse exists if the domain is restricted to positive values of $x$.

## Even and odd functions

### Odd

A function $f(x)$ is odd if $f(-x) = -f(x)$ for every $x$ in its domain. For example, $f(x) = x^3$ is odd because $(-x)^3 = -x^3$. The graph of an odd function is symmetric with respect to the origin. The graph for negative values of $x$ can be obtained by rotating the graph for positive values of $x$ through 180° about the origin.

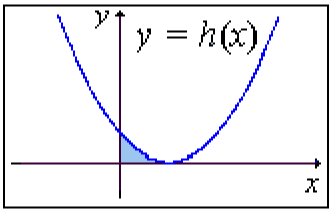
### Even

A function $f(x)$ is even if $f(-x) = f(x)$ for every $x$ in its domain. For example, $f(x) = x^2$ is even because $f(-x) = (-x)^2 = x^2 = f(x)$. Note that the graph of an even function is symmetric with respect to the $y$-axis.

## Examples:

Are the following functions even, odd or neither?

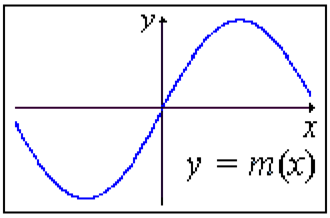
1



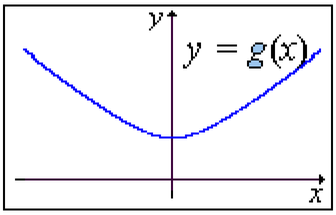

◯ Even ◯ Odd ◯ Neither

2

◯ Even ◯ Odd ◯ Neither




3




◯ Even ◯ Odd ◯ Neither

STRATEGIE DI SOSTEGNO ALLA COMPRENSIONE (INPUT COMPRENSIBILE)

**Ricordarsi ripetere e di enfatizzare i punti importanti per distinguerli da aspetti secondari/dettagli.**

Utilizzo di:

a. glossario microlingua specifica
b. grafico

# GLOSSARY

| | | |
|---|---|---|
| N;Z;Q;R | = | Set of Natural, Integer, Rational, Real numbers |
| a | = | element |
| : or / | = | such that |
| $a \in N$ | = | a belongs to N ( a is an element of N ) |
| $a \notin N$ | = | a does not belong to N |
| $a = b$ | = | a equals b ( a is equal to b) |
| $a \neq b$ | = | a different from b |
| $a + b$ | = | a plus b |
| $a - b$ | = | a minus b |
| $a \cdot b$ | = | a times b |
| $a : b$ | = | a divided by b |
| $a / b$ | = | a over b |
| 2/3 | = | two thirds ( two over three) |
| 1/3 | = | one third |
| 3/ 4 | = | three quarters |
| 1/ 2 | = | one half (a half) |
| -3 | = | minus three ( the opposite of plus three ) |
| $a < b$ | = | a less than b |
| $a \leq b$ | = | a less than or equal to b |
| $a > b$ | = | a greater than b |
| $a \geq b$ | = | a greater than or equal to b |
| $a > 0$ | = | a greater than zero ( a is a positive number ) |
| $a < 0$ | = | a less than zero ( a is a negative number ) |
| 2, 3, 5, 7,11… | = | prime numbers |
| 0, 2, 4, 6, 8… | = | even numbers |
| 1, 3, 5, 7, 9, … | = | odd numbers |
| 0 ,1 ,4 ,9 ,16 , 25… | = | square numbers |
| $a^n$ | = | a raised to the power n ( a to the n, a to the nth ) |
| $a^2$ | = | a squared ( the square of a) |
| $a^3$ | = | a cubed ( the cube of a , a to the power three ) |
| ( ) | = | bracket |
| [ ] | = | square bracket |
| { } | = | brace |
| unknown | = | incognita |
| variable | = | variabile |
| left hand side | = | primo membro (membro a sinistra ) |

| | | |
|---|---|---|
| right hand side | = | secondo membro ( membro a destra ) |
| to add to | = | addizionare a |
| to subtract from | = | sottrarre a |
| to multiply by | = | moltiplicare per |
| to divide by | = | dividere per |

| | | |
|---|---|---|
| Axis | = | asse |
| Ordered pair | = | Coppia ordinata |
| Cartesian product AxB | = | Prodotto cartesiano AxB |
| Cartesian plane | = | piano cartesiano |
| Degree | = | grado |
| Equation | = | equazione |
| Function | = | funzione |
| Gradient | = | gradiente,coefficiente angolare della retta y=mx+q |
| Graph | = | grafico |
| Half-plane | = | semipiano |
| Horizontal axis | = | asse delle ascisse (x-axis ) |
| Inequality | = | disequazione |
| Intercept | = | intercetta,termine noto q nell'equazione della retta y=mx+q |
| Linear | = | lineare,di primo grado |
| Parallel | = | parallela |
| Plane | = | piano |
| Slope | = | pendenza |
| Solution | = | soluzione |
| Solution set | = | insieme delle soluzioni |
| Straight line | = | retta |
| Vertical axis | = | asse delle ordinate ( y-axis ) |

Oss.: altri termini propri del modulo vengono spiegati di volta in volta sulle fotocopie fornite agli allievi durante le lezioni

III) STRATEGIE PER LA VERIFICA COMPRENSIONE IN ITINERE.

**Verifica della comprensione dopo ogni segmento significativo di attività.**
( seconda parte)

Class 3 Eli

Fill in the gaps:

1. Let f: A→B and g: B→C be two functions; the ………………… ………………. g°f : …. →….. is a function defined as (g°f )(x)= g(….. for all x taken in the set ……

2. The graph of a ………….. f: A→B is the set of all …………….. pairs in the form ( ….. , …….. ) where x………. and …….. $\in B$

3. The horizontal line test is useful to find out if the graph …………………………………….

4. The vertical line test is useful to find out if the graph …………………………………….

5. f: A→B is invertible if …………………………………………….

6. f: A→B is an ……….. function if f(-x)=f(x) for all x taken in the set …… and its …………..is symmetrical about the …. ………

7. f: A→B is an ……….. function if f(-x)= -f(x) for all x taken in the set …… and its …………..is symmetrical about the …………..

IV) HOMEWORK

**Le attività sopra elencate possono anche essere scelte come compiti per casa. Si assegnano degli esercizi finalizzati al consolidamento della comprensione dei contenuti ( esercizi del libro di testo)e dell'acquisizione della microlingua (studio delle definizioni e vocaboli specifici).**

V) SOSTEGNO ALLA PRODUZIONE ORALE/SCRITTA DEGLI STUDENTI

Mediante:

a. glossario specifico
b. testi da completare
c. frase iniziale di un paragrafo
d. lettura ed interpretazione di un grafico
e. punti da discutere ( a coppie o a gruppi) *problem-solving*
f. schema ( punti) per lavoro di gruppo/a coppia
g. lessico/ strutture/*phrases* essenziali per presentare un argomento
h. risoluzione di esercizi guidati
i. questions loop( allegato)

# VERIFICA E VALUTAZIONE

Name………………………………………….                    Class 3 Eli A

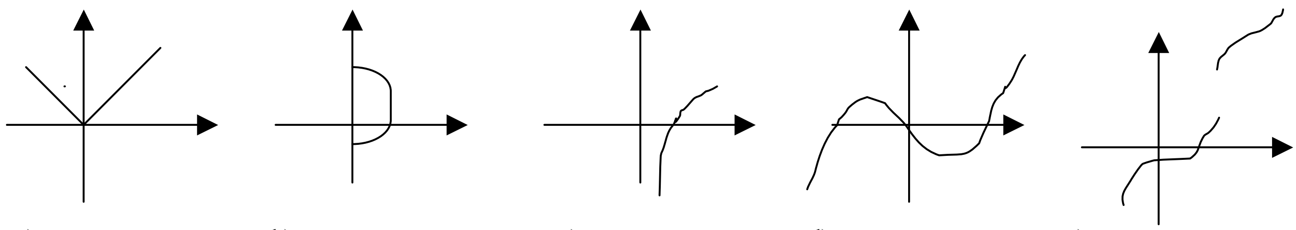
TEST

**Don't forget to give reasons to your answers**

1. Explain the difference between MAP and FUNCTION . (max 4 lines)

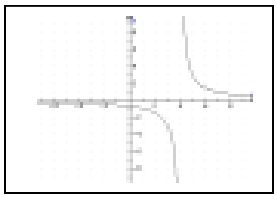
2. Give the definition of " injective function".

3. Decide if the following graphs represent functions, from the domain to the real number set, or not. For each function state the type.



a)          b)          c)          d)          e)

4. The diagram shows the graph of the function $f(x) = \dfrac{3}{x-4}$ .



a) Its domain D is :   * R   * R- $\{4\}$   * R- $\{-4\}$   * $\{x \in R : x > 4\}$

b) Its range f(D) is :   * R   * R- $\{0\}$   * R- $\{4\}$   * $\{x \in R : y > 0\}$

c) This function f: D→ f(D) is:   * just a function   * bijective   * surjective   * injective

d) Its inverse function is …………….. ……

e) Can you draw the graph? ( use different colour )

5. Let f: N→N , x→2x+3
   a) find the range f(N) and sketch the graph.( Pay attention !!)

   b) Is f a surjective function? Why?………………………………………………………………………………………………

6. Let f: N→ Z, x→$x^2$-4x . Find the inputs that have -4 , 0 or 5 as images.
   Can you use the previous answers to state that f is not one-to-one ? In which way?

7. Find the domain of $f(x) = \sqrt{|x| - x - 1}$ .

8. Decide if the following functions are EVEN, ODD or NEITHER: f(x)=$x^2$-2x ; g(x)= -5$x^3$+7x ; h(x)= $\dfrac{4}{x^2-3}$ ; y=2x+3

9. Have the functions $f(x) = \dfrac{x^2-16}{x-4}$ and $g(x) = x + 4$ the same domain?…………Are these functions equal?
   Why?……………………………………..…………………………………………………………………………………..…

Name……………………………………… Class 2 Est Date…………

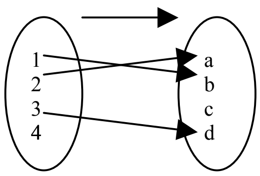
FUNCTIONS

TEST

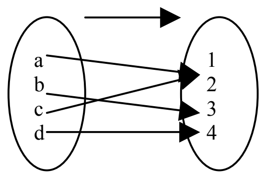
1. A cartesian product AxB is ………………

2. A function is bijective if………

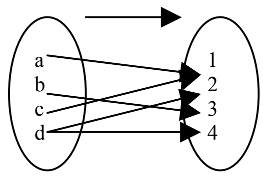
3. Determine whether the below relations from A to B are functions or not and fill in the gaps :




a) f : A→ B




b) g : A→ B




c) h : A→ B

f …………..a function because the element d is ……………………

Domain of f = ……………; Range of f =………………;

g …………..a function because no elements of ….. are mapped to more than …… in ……

Domain of g is ……………………; Range of g is ………………;

h …………..a function because ……………………………………..

Domain of h is ……………; Range of h is ………………;

4. The relation $R$ :$R^+$→ R is defined : x $R$ y ⇔ $y^2 = 4x$ . Is $R$ a function? Why?
   Write at least three pairs of $R$

5. Let A= $\{1,2,3,4,-2\}$ be a set and $T$ : A→ A the relation that pairs two components $x \in A$ and $y \in A$ only if $\frac{y}{x} \in N$. Write the elements of the relation $T$ in list form. Write **domain** and **range** too.
   Is $T$ a function ? Why?

Questions loop

| | | | |
|---|---|---|---|
| A | The cartesian product AxB is | B | Is the set of real numbers greater than zero or equal to zero |
| A | What's an ordered pair ? | B | The set of all ordered pairs ( a , b ) where a is taken in the set A and b (is taken) in the set B |
| A | A map from A to B | B | A pair in the form (a , b) where a is the first component and b the second component |
| A | A map can also be called | B | Is a subset of AxB |
| A | What's a function A→ B ? | B | Relation or mapping |
| A | In a function the set of all elements that have an image is called | B | It is a rule that maps each element in A onto exactly one element in B |
| A | The Range of a function is | B | Domain |
| A | Functions are mappings of the type | B | The set of all images |
| A | A map is one-to-one | B | One-to-one or many-to-one |
| A | A map is many-to-one | B | If each one value x maps onto exactly one value of y |
| A | A function is injective | B | If two or more values of x map onto exactly one value of y |
| A | A function from A to B is surjective | B | If different elements of the domain are matched with different elements of the range |
| A | If every element of the set B is the image of at least one element of the domain ,the function from A to B is | B | If the range is equal to B |

| | | | |
|---|---|---|---|
| A | A functions is bijective | B | Surjective or onto |
| A | What is the vertical line test useful for? | B | If it is both injective and surjective |
| A | To prove if a graph is the graph of an injective function | B | To prove if a graph represents a function or not |
| A | If every vertical line intersects a graph only in a point | B | It's possible to use the horizontal line test |
| A | A function is invertible | B | The graph represents a function |
| A | The function y = 2x , from N to N | B | If it is bijective |
| A | The function y = 2x , from N to the set of even numbers | B | Is injective but not surjective |
| A | The range of the function $y = x^2$ ,from R to R, | B | Is both injective and surjective , that means it a bijective function |